

DM551 Peter Serein Rasmussen

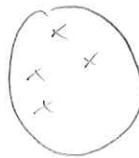
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DM551 | 1 | 09-09-16

Product rule :	p. 376
Sum rule :	p. 379
Inc-excl :	p. 382
php :	p. 388
gen php :	p. 390
Thm 6.2.3 :	p. 392
Ramsey :	p. 393

- Intel kontor
- PDF ≠ international/global ⇒ wrong exercises

6.1 18 math stud 325 CS stud ✓



a) # ways : Two repr - one from each? $18 \cdot 325 = 5850$

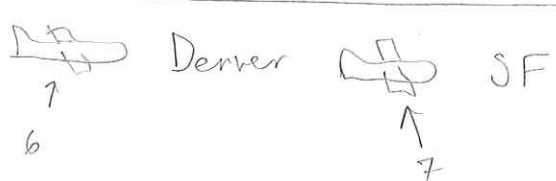
b) # ways : one repr - one from either? $18 + 325 = 343$

6.1.2 10 Qs, 4 choices (Multiple choice test) ✓

a) # ways to answer test? $\underbrace{4 \cdot 4 \cdot 4 \cdots 4}_{10} = 4^{10}$

b) # New choice: leave blank. # ways? $\underbrace{5 \cdot 5 \cdots 5}_{10} = 5^{10}$

6.1.3 NY Denv SF ✓



$6 \cdot 7 = 42$



6.1.16 License plate: $XX/XXX + YY/YYYY$

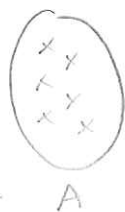
X: char (26)
Y: digit (10)

Div in cases

$$\begin{aligned} & XXYY: 26^2 \cdot 10^2 \\ & + XXXYY: 26^3 \cdot 10^2 \\ & + XXYYY: 26^2 \cdot 10^3 \\ & + XXXYYY: 26^3 \cdot 10^3 \end{aligned}$$

6.1.18 $f: A \rightarrow B$

$|A| = 10$



a) $|B| = 2$



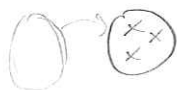
2 choices



2 choices

$$\underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{10} = 2^{10}$$

b) $|B| = 3$



$$3 \cdot 3 \cdot 3 = 3^{10}$$

c) $|B| = 4$

$$4 \cdot 4 \cdot 4 \dots 4 = 4^{10}$$

d) $|B| = 5$

$$5 \cdot 5 \cdot 5 = 5^{10}$$

"e)" $|B| = m$

$$m^{10}$$

"f)"

$$|A| = n$$

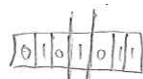
$$|B| = m$$

$$m^n$$

6.1.25 # diff bit str palindrome of len n ?

▷ # bit str len n ? 2^n

▷ Palindrome?



▷ Palindrome bit str?

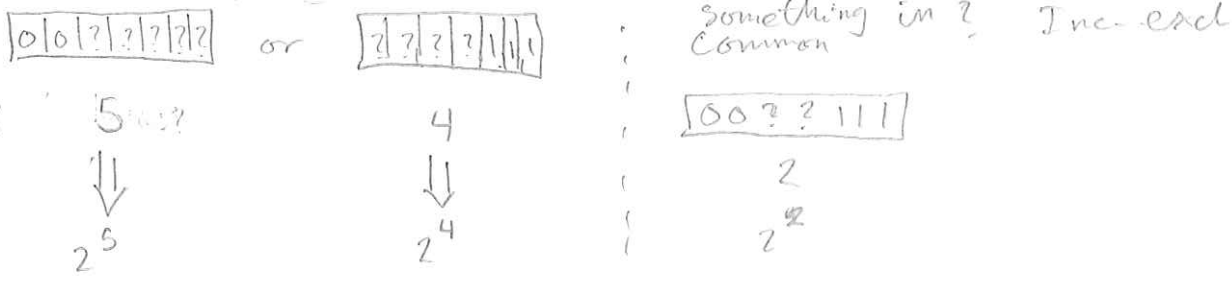
How many choices?

$$n/2 \text{ (even)} \Rightarrow 2^{n/2}$$

$$n/2 + 1 \text{ (odd)} \Rightarrow 2^{n/2 + 1}$$

$$\Rightarrow 2^{\lceil n/2 \rceil}$$

6.1.32 # diff bit str: begin w. 00 or end w. 111 ✓



Result: $2^5 + 2^4 - 2^2$

6.1.44 p, q prime, $n = p \cdot q$ ✓

▷ Nums, $1, \dots, n$
 Find those relative prime

Side note:
 ▷ $\varphi(n)$
 ▷ $\varphi(p \cdot q) = (p-1)(q-1)$
 ▷ $p \cdot q - (p-1)(q-1) = p+q-1$

▷ Relative prime? $\gcd(a, b) = 1$
 i.e. no common factors

▷ How can we (as a number) have common factors with n ?
 Must be multiple of p or q



Answer seems $p+q$

▷ Inc-excl: $q \cdot p$ and $p \cdot q$
 so $p+q-1$

▷ Final answer $p \cdot q - (p+q-1)$

6.1.48 Product rule: $a_1 \cdot a_2$ expand to $a_1 \cdot a_2 \cdot \dots \cdot a_n$; Induction ✓

▷ Induction: $P(n) = a_1 \cdot a_2 \cdot \dots \cdot a_n$, bases, ind hyp, ind step

▷ Bases: $P(2) = a_1 \cdot a_2$, trivial

▷ Ind. step: Assume $P(n)$, show $P(n+1) = a_1 \cdot a_2 \cdot \dots \cdot a_n \cdot a_{n+1}$

By ind hyp $a_1 \cdot a_2 \cdot \dots \cdot a_n = b$ becomes correct amount of choices/ways
 $b \cdot a_{n+1}$ by product rule is new #ways

6.2.2 30 stud in class, show $\exists 2$ with same first letter of last name.


▷ # first letter of last name = 26 (boxes)



if $N > K$: some box contains > 1 obj

▷ Obj's? 30 stud

▷ Boxes? 26 letter

6.2.4 *  $\begin{matrix} \text{red} \\ \text{blue} \end{matrix} \times 10$ woman select ball random (but we don't deal with probs yet)

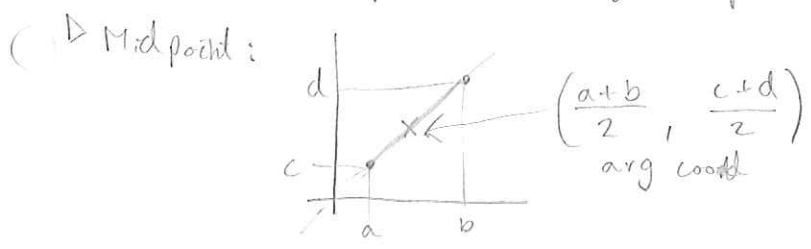
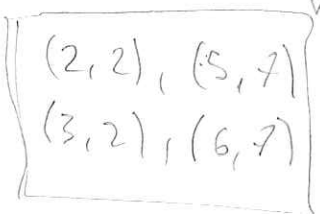
a) How many before $\text{red} \times 3$ or $\text{blue} \times 3$?

▷ Go for red , try to get blue instead. Get $\text{blue} \times 2$, then $\text{red} \times 2$ next will fulfill req $\Rightarrow 5$

b) How many before $\text{red} \times 3$ (or symmetrically $\text{blue} \times 3$)

▷ Get all $\text{blue} \times 10$ first, then $\text{red} \times 3 \Rightarrow 13$

6.2.8 Given 5 points: $(x_i, y_i), i=1 \dots 5$, distinct, xy-plane. Show $\exists 2$ points with integer midpoint.



▷ When to expect int-coords? If $a+b$ even and $c+d$ even

▷ How can $a+b$ be even? If a even, b even or a odd, b odd; $\frac{2}{4}$ outcomes some parity

▷ When are both coord even?



6.2.14 9 stud (male/female) ✓

a) Show ≥ 5 male or ≥ 5 female

▷ Gen. pph: $\begin{matrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & & \end{matrix} \rightarrow \begin{matrix} \square & \square & \square \\ \square & \square & \square \\ \square & & \end{matrix} \Rightarrow$ some box has $\lceil \frac{N}{K} \rceil$ elem

N obj K boxes

▷ objs? 9 stud = N $\lceil \frac{N}{K} \rceil = \lceil \frac{9}{2} \rceil = 5$
 boxes? : 2 Genders = K

b) Show ≥ 3 male or ≥ 7 female (skewed pph)

▷ Assume ≤ 2 male $\Rightarrow 9 - 2 = 7$ female

▷ Assume ≤ 6 female $\Rightarrow 9 - 6 = 3$ male

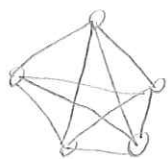
6.2.18 101 standing in line, diff height, Show 11 in inc order ^(✓)
 order

▷ Use Thm 3 p. 392: "Every seq of $n^2 + 1$ contains subseq of len $n+1$ in inc or dec order"

▷ what is n ? $n = 10$
 $(n^2 + 1 = 10^2 + 1 = 101$ and $n + 1 = 10 + 1 = 11)$

6.2.20 Show in group of 5 people (which can be friends or foes) ^(✓)
 there are not 3 mutual friends/foes (as opposed to 6 people, Ramsey, $R(3,3)=6$)

▷ Consider



assign among or $\circ \cdots \circ$
 foe friend



consider each triangle will be



6.2.26 $> 800k$ ppl in Paris, assumed $\leq 200k$ hair on head
 each $\neq \dots 200k$ is possible
 a) show some # of hair on head
 \triangleright PHP? Obj? $800k$ ppl
 Boxes? $200k$ diff # of hair

b) show 5 with some # of hair on head

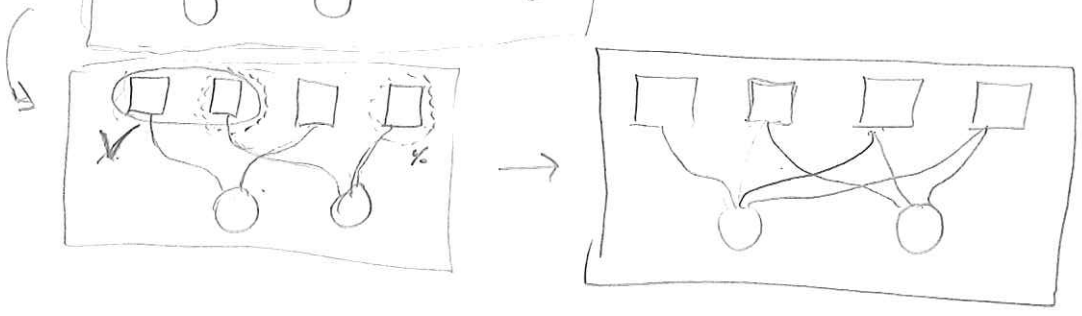
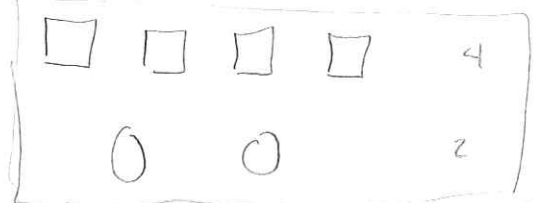
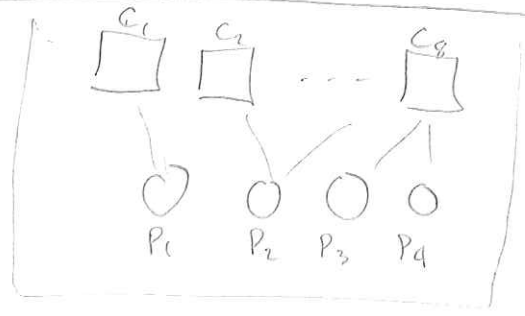
$N \geq 800k + 1$

$k = 200k$

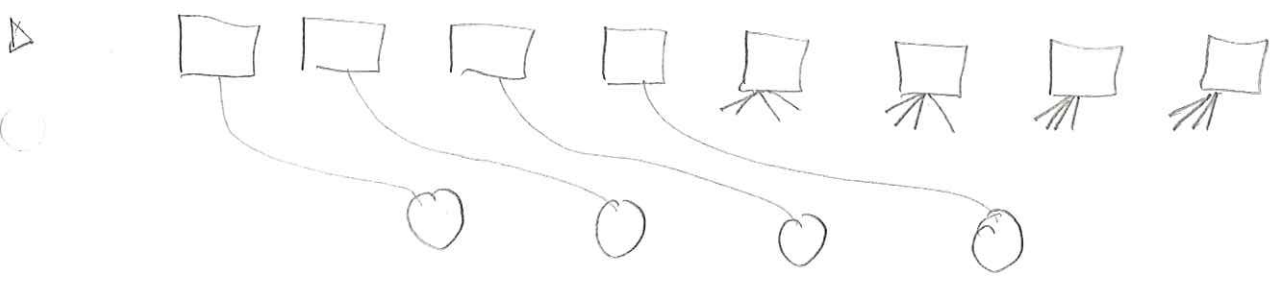
$$\left\lceil \frac{800k+1}{200k} \right\rceil = 5$$

6.2.30 8 computers, 4 printers


\triangleright To understand problem, consider:



$2 + 2 \cdot 2 = 6$



$4 + 4 \cdot 4 = 20$

6.2.35 * 51 houses  on street, addresses 1000-1099

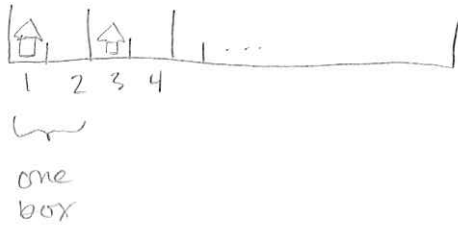
⇒ 100 addresses

Show 2 houses on consecutive addresses, eg. 42, 43.

▷ gen. prob, obj? 51 x 

▷ what about boxes?

Consider



if 2 in same box ⇒
consecutive addresses.

How many boxes? 50 x 

$$N = 51$$

$$k = 50$$

$$\left\lceil \frac{51}{50} \right\rceil = 2$$